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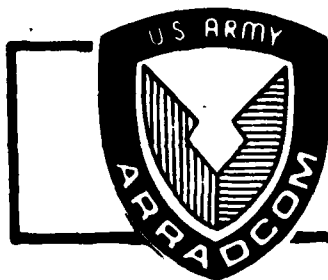
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TECHNICAL REPORT ARLCB-TR-81038

ELASTIC-PLASTIC THICK-WALLED TUBES SUBJECTED TO  
INTERNAL PRESSURE AND TEMPERATURE GRADIENT

P. C. T. Chen

September 1981



US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND  
LARGE CALIBER WEAPON SYSTEMS LABORATORY  
BENET WEAPONS LABORATORY  
WATERVLIET, N. Y. 12189

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## INTRODUCTION

The elastic-plastic problems of thick-walled tubes subjected to mechanical loadings have been solved by many investigators based on different theories or methods.<sup>1-4</sup> Very little work has been done on the elastoplastic solutions for thick-walled tubes subjected to thermal loadings.<sup>5-7</sup> Recently, a new finite-difference approach was developed for solving the plane-strain problems of elastic-plastic thick-walled tubes subjected to mechanical loadings<sup>3</sup> or thermal loadings.<sup>7</sup> The approach has been extended to solve the generalized plane-strain problem subjected to mechanical loadings.<sup>4</sup>

In the present report, the generalized plane-strain problems of elastic-plastic thick-walled tubes subjected to mechanical as well as thermal loadings are considered. The formulation includes internal pressure, external pressure, axial force, steady or transient thermal loadings. The numerical result is reported for a closed-end tube subjected to internal pressure and to temperature gradient. The formulation is based on the incremental finite-difference method using von Mises' criterion, the Prandtl-Reuss flow theory and the isotropic hardening rule. All incremental quantities are determined in the program and no iteration is needed. In order to improve the efficiency of the program, a scaled loading approach has been implemented. The numerical results have been compared with those of Bland<sup>5</sup> and additional results due to large temperature gradient are reported.

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References are listed at the end of this report.

## BASIC EQUATIONS

Assuming small strain and no body forces in the axisymmetric state of generalized plane strain, the radial and tangential stresses,  $\sigma_r$  and  $\sigma_\theta$ , must satisfy the equilibrium equation,

$$r(\partial\sigma_r/\partial r) = \sigma_\theta - \sigma_r ; \quad (1)$$

and the corresponding strains,  $\epsilon_r$  and  $\epsilon_\theta$ , are given in terms of the radial displacement,  $u$ , by

$$\epsilon_r = \partial u / \partial r , \quad \epsilon_\theta = u / r . \quad (2)$$

It follows that the strains must satisfy the equation of compatibility

$$r(\partial\epsilon_\theta/\partial r) = \epsilon_r - \epsilon_\theta . \quad (3)$$

Whereas the differential equations (1), (2), and (3) hold throughout the tube regardless of the material properties, the constitution equations assume various forms according to the adopted form of yield function, hardening rule, total or incremental theory of plasticity. In the present report, the material is assumed to be elastic-plastic, obeying the von Mises' yield criterion, the Prandtl-Reuss flow theory and the isotropic hardening law. The complete stress-strain relations are:

$$\Delta\epsilon_i' = \Delta\sigma_i' / 2G + (3/2)\sigma_i' \Delta\sigma / (\sigma H') \quad (4)$$

$$\Delta\sigma > 0 \quad \text{for } i = r, \theta, z$$

$$\Delta\epsilon_m = E^{-1}(1-2\nu)\Delta\sigma_m + \alpha\Delta T \quad (5)$$

where  $E$ ,  $\nu$ ,  $\alpha$  are Young's modulus, Poisson's ratio, coefficient of thermal expansion, respectively,  $\Delta T$  is the temperature increment,

$$2G = E/(1+\nu) ,$$

$$\epsilon_m = (\epsilon_r + \epsilon_\theta + \epsilon_z)/3 , \quad \epsilon_1' = \epsilon_1 - \epsilon_m ,$$

$$\sigma_m = (\sigma_r + \sigma_\theta + \sigma_z)/3 , \quad \sigma_1' = \sigma_1 - \sigma_m ,$$

$$\sigma = (1/\sqrt{2})[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2]^{1/2} > \sigma_0 , \quad (6)$$

and  $\sigma_0$  is the yield stress in simple tension or compression. For a strain-hardening material,  $H'$  is the slope of the effective stress/plastic strain curve

$$\sigma = H(\int d\epsilon^p) . \quad (7)$$

For an ideally-plastic material ( $H' = 0$ ), the quantity  $(3/2)d\sigma(\sigma H')$  is replaced by  $d\lambda$ , a positive factor of proportionality. When  $\sigma < \sigma_0$  or  $d\sigma < 0$ , the state of stress is elastic and the second term in equation (4) disappears. Following Yamada et al,<sup>8</sup> equations (4) and (5) can be rewritten in an incremental form

$$\Delta\sigma_1 = d_{1j}\Delta\epsilon_j - E\alpha\Delta T/(1-2\nu) \quad \text{for } i,j = r,\theta,z$$

and

$$d_{1j}/2G = \nu/(1-2\nu) + \delta_{1j} - \sigma_1'\sigma_j'/S , \quad (8)$$

where

$$S = \frac{2}{3} \left(1 + \frac{1}{3} H'/G\right) \sigma^2 , \quad H'/E = \omega/(1-\omega) , \quad (9)$$

$\omega E$  is the slope of the effective stress-strain curve, and  $\delta_{1j}$  is the Kronecker delta.

Consider an open-end or closed-end thick-walled tube of inner radius  $a$  and external radius  $b$ . The tube is subjected to inner pressure  $p$ , external pressure  $q$ , and force  $f$ , inner temperature  $T_a$  and external temperature  $T_b$ . The boundary conditions for the problem are



$$\sigma_r(a,t) = -p, \quad \sigma_r(b,t) = -q,$$

$$2\pi \int_a^b r \sigma_z dr = \mu \pi a^2 p + f \quad (10)$$

where  $\mu$  is 0 for open-end tubes and 1 for closed-end tubes.

When the temperature  $T$  is not varying with respect to time, the steady state distribution is given by

$$T = T_a + N \log (r/a)$$

and

$$N = (T_b - T_a) / \log (b/a) \quad (11)$$

The stress solution in the elastic range is well-known. The quantities  $p^*$ ,  $q^*$ ,  $f^*$ ,  $T_a^*$  or  $T_b^*$  required to cause initial yielding can be determined by using the von Mises' yield criterion.

#### FINITE-DIFFERENCE FORMULATION

For loading beyond the elastic limit, an incremental approach of the finite-difference formulation is used. The cross section of the tube is divided into  $n$  rings with  $r_1=a, r_2, \dots, r_k=\rho, \dots, r_{n+1}=b$ , where  $\rho$  is the radius of the elastic-plastic interface. At the beginning of each increment of loading, the distribution of temperature, displacements, strains, and stresses is assumed to be known and we want to determine  $\Delta u$ ,  $\Delta \epsilon_r$ ,  $\Delta \epsilon_\theta$ ,  $\Delta \epsilon_z$ ,  $\Delta \sigma_r$ ,  $\Delta \sigma_\theta$ ,  $\Delta \sigma_z$  at all grid points for the applied incremental loading,  $\Delta p$ ,  $\Delta q$ ,  $\Delta f$ ,  $\Delta T_i$  ( $i = 1$  to  $n+1$ ). Since the incremental stresses are related to the incremental strains by the incremental form (Eq. (8)) and  $\Delta u = r \Delta \epsilon_\theta$ , there exists only three unknowns at each station that have to be determined for each increment of loading. Accounting for the fact that the axial strain  $\epsilon_z$  is independent of  $r$ , the unknown variables in the present formulation are  $(\Delta \epsilon_\theta)_i$ ,  $(\Delta \epsilon_r)_i$ , for

$i = 1, 2, \dots, n, n+1$ , and  $\Delta \epsilon_z$ .

The equation of equilibrium (1) and the equation of compatibility (3) are valid for both the elastic and the plastic regions of a thick-walled tube. The finite-difference forms of these two equations at  $i = 1, \dots, n$  are given by

$$\begin{aligned} (r_{i+1}-2r_i)(\Delta \sigma_r)_i - (r_{i+1}-r_i)(\Delta \sigma_\theta)_i + r_i(\Delta \sigma_r)_{i+1} \\ = (r_{i+1}-r_i)(\sigma_\theta - \sigma_r)_i - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \end{aligned} \quad (12)$$

for the equation of equilibrium, and

$$\begin{aligned} (r_{i+1}-2r_i)(\Delta \epsilon_\theta)_i - (r_{i+1}-r_i)(\Delta \epsilon_r)_i + r_i(\Delta \epsilon_\theta)_{i+1} \\ = (r_{i+1}-r_i)(\epsilon_r - \epsilon_\theta)_i - r_i[(\epsilon_\theta)_{i+1} - (\epsilon_\theta)_i] \end{aligned} \quad (13)$$

for the equation of compatibility. With the aid of the incremental stress-strain relations (Eq. (8)), equation (12) can be rewritten as

$$\begin{aligned} [(r_{i+1}-2r_i)(d_{12})_i + (-r_{i+1}+r_i)(d_{22})_i](\Delta \epsilon_\theta)_i \\ + [(r_{i+1}-2r_i)(d_{11})_i + (-r_{i+1}+r_i)(d_{21})_i](\Delta \epsilon_r)_i \\ + r_i(d_{12})_{i+1}(\Delta \epsilon_\theta)_{i+1} + r_i(d_{11})_{i+1}(\Delta \epsilon_r)_{i+1} \\ + [(r_{i+1}-2r_i)(d_{13})_i + (-r_{i+1}+r_i)(d_{23})_i + r_i(d_{13})_{i+1}]\Delta \epsilon_z \\ = (r_{i+1}-r_i)(\sigma_\theta - \sigma_r)_i - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \\ + r_i E\alpha(1-2\nu)^{-1}(\Delta T_{i+1} - \Delta T_i) \end{aligned} \quad (14)$$

The boundary conditions for the problem are

$$\begin{aligned} \Delta \sigma_r(a, t) = -\Delta p, \quad \Delta \sigma_r(b, t) = -\Delta q, \\ \pi \sum_{i=1}^n [r_i(\Delta \sigma_z)_i + r_{i+1}(\Delta \sigma_z)_{i+1}](r_{i+1}-r_i) = \mu \pi a^2 \Delta p + \Delta f, \end{aligned} \quad (15)$$

where  $\mu$  is 0 for open-end tubes and 1 for closed-end tubes. Using the incremental relations (Eq. (8)), we rewrite equation (15) as

$$(d_{12})_1(\Delta\epsilon_\theta)_1 + (d_{11})_1(\Delta\epsilon_r)_1 + (d_{13})_1\Delta\epsilon_z = -\Delta p + E\alpha(1-2\nu)^{-1}\Delta T_1 \quad (16)$$

$$(d_{12})_{n+1}(\Delta\epsilon_\theta)_{n+1} + (d_{11})_{n+1}(\Delta\epsilon_r)_{n+1} + (d_{13})_{n+1}\Delta\epsilon_z = -\Delta q + E\alpha(1-\nu)^{-1}\Delta T_{n+1} \quad (17)$$

and

$$\begin{aligned} & \sum_{i=1}^n (r_{i+1}-r_i) \{ r_i [(d_{23})_i(\Delta\epsilon_\theta)_i + (d_{13})_i(\Delta\epsilon_r)_i] + r_{i+1} [(d_{23})_{i+1}(\Delta\epsilon_\theta)_{i+1} \\ & + (d_{13})_{i+1}(\Delta\epsilon_r)_{i+1}] \} + \sum_{i=1}^n (r_{i+1}-r_i) [r_i(d_{33})_i + r_{i+1}(d_{33})_{i+1}] \Delta\epsilon_z \\ & = \mu a 2\Delta p + \Delta f/\pi + \sum_{i=1}^n (r_{i+1}-r_i) [r_i\Delta T_i + r_{i+1}\Delta T_{i+1}] E\alpha/(1-2\nu) \quad (18) \end{aligned}$$

Now we can form a system of  $2n+3$  equations for solving  $2n+3$  unknowns,  $(\Delta\epsilon_\theta)_i$ ,  $(\Delta\epsilon_r)_i$ , at  $i = 1, 2, \dots, n, n+1$  and  $\Delta\epsilon_z$ . Equations (16), (17), and (18) are taken as the first and last two equations, respectively, and the other  $2n$  equations are set up at  $i = 1, 2, \dots, n$  using equations (13) and (14). The final system is an unsymmetric matrix of arrow type with the nonzero terms appearing in the last row and column and others clustered about the main diagonal, two below and one above.

#### INCREMENTAL LOADING - FIXED VS. SCALED

When the total applied pressure  $p$  or temperature  $T_i (i=1 \text{ to } n+1)$  is given, it is natural to divide the loading path into  $m$  equal fixed increments such as  $\Delta p = (p-p^*)/m$ ,  $\Delta T_i = (T_i-T_i^*)/m$ . These fixed increments need not be equal for all steps and any sequence of  $m$  increments can be supplied by the user. A sequence of decreasing load-increments is a better choice than that of equal increments.

In order to increase the efficiency of the program, an adaptive algorithm based on a scaled incremental-loading approach<sup>8</sup> has been implemented. In each step, a dummy load-increment such as  $\Delta p$  is applied and the incremental results  $\Delta \sigma_i$  for  $i = r, \theta, z$  at all grids are determined. For all grid points at which  $\sigma = ||\sigma_i|| < \sigma_0$ , we compute the scaler  $\alpha$ 's by the formula

$$\alpha = \frac{1}{2} \{ \Gamma + [\Gamma^2 + 4 ||\Delta \sigma_i||^2 (\sigma_0^2 - ||\sigma_i||^2)]^{1/2} / ||\Delta \sigma_i||^2 \}, \quad (19)$$

where

$$\Gamma = ||\sigma_i||^2 + ||\Delta \sigma_i||^2 - ||\sigma_i + \Delta \sigma_i||^2, \quad (20)$$

and  $||\sigma_i||^2$ ,  $||\Delta \sigma_i||^2$ ,  $||\sigma_i + \Delta \sigma_i||^2$  are computed by

$$||\sigma_i||^2 = \frac{1}{2} [(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2]. \quad (21)$$

Let  $\lambda$  be the minimum of the  $\alpha$ 's. Then  $\lambda$  is the load-increment factor just sufficient to yield one additional point. A sequence of  $\lambda(j)$  can be determined for all steps  $j = 1, 2, \dots, m$  and the updated results are

$$\begin{aligned} p(j) &= p(j-1) + \lambda(j) \Delta p(j) \\ \sigma_i(j) &= \sigma_i(j-1) + \lambda(j) \Delta \sigma_i(j), \text{ etc.} \end{aligned} \quad (22)$$

#### NUMERICAL RESULTS AND DISCUSSIONS

Consider a closed-end tube subjected to internal pressure  $p$ , inner and outer temperature  $T_a$  and  $T_b$ . The numerical results were based on the following parameters:  $b = 2"$ ,  $a = 1"$ ,  $n = 100$ ,  $\nu = 0.3$ ,  $E = 30 \times 10^6$  psi,  $\sigma_0 = 30 \times 10^3$  psi,  $\omega = 0$ ,  $\alpha = 7.75 \times 10^{-6}$  in/in°F.

According to Bland,<sup>5</sup> let us define

$$\theta = E \alpha n / [2(1-\nu) \sigma_0 / \sqrt{3}]$$

as a measure of the effect of the temperature differences in the stress system. The mean temperature

$$T_m = \frac{2(b^2 - a^2)^{-1}}{2} \int_a^b T r dr$$

is taken as zero in the calculation of  $u/r$ . Three values of the temperature stress factor  $\theta$  are chosen:  $\theta = -1/2, 0$ , and  $1/2$ , i.e.,  $T_b - T_a = -72.3^\circ\text{F}$ ,  $0^\circ\text{F}$ , and  $72.3^\circ\text{F}$ . The thermal stresses are in the elastic range. In the presence of these temperature gradients, internal pressure  $p$  is applied incrementally until the fully plastic state is reached. The internal pressure  $p$  and inside displacement  $U_a$  are obtained as functions of elastic-plastic interface  $\rho$  as shown in Figure 1. The effect of temperature gradient on these relations is clearly shown in the figure. In order to compare the results by Bland,<sup>5</sup> the state of stress is evaluated at  $\rho/a = 1.73$ . The dimensionless stresses  $(\sigma_r, \sigma_\theta, \sigma_z)/\sigma_0$  are expressed as functions of  $r/a$  as shown in Figures 2 through 4 for three cases of  $\theta = 0, -1/2, 1/2$ , respectively.

After removing the temperature gradients and internal pressure, the residual stresses were obtained for all three cases. The states of residual stresses for the first two cases were still elastic but reverse yielding occurred when unloading the last case. Five scaled incremental-loading steps and one applied incremental-loading step were needed to unload completely. The residual stresses for the last case are shown in Figure 5. The results in Figures 1 through 5 are roughly the same as those of Bland<sup>5</sup> except  $\sigma_z$ . This difference is reasonable because Tresca's yield criterion is used in reference 5.

As a last example let us consider a closed-end tube subjected to inner temperature  $T_a$  only. When the temperature gradient is of sufficient magnitude, yielding will first expand from the inside. When  $T_a$  is larger than a certain limit ( $238.4^\circ\text{F}$ ), plastic zone will expand from both the inside and outside surface toward the interior. The relation between the inside temperature and elastic-plastic interface is shown in Figure 6. The stresses in a closed-end tube subjected to temperature gradient ( $T_a = 299^\circ\text{F}$ ,  $T_b = 0$ ) are shown in Figure 7. The dotted lines are elastic-plastic interfaces.

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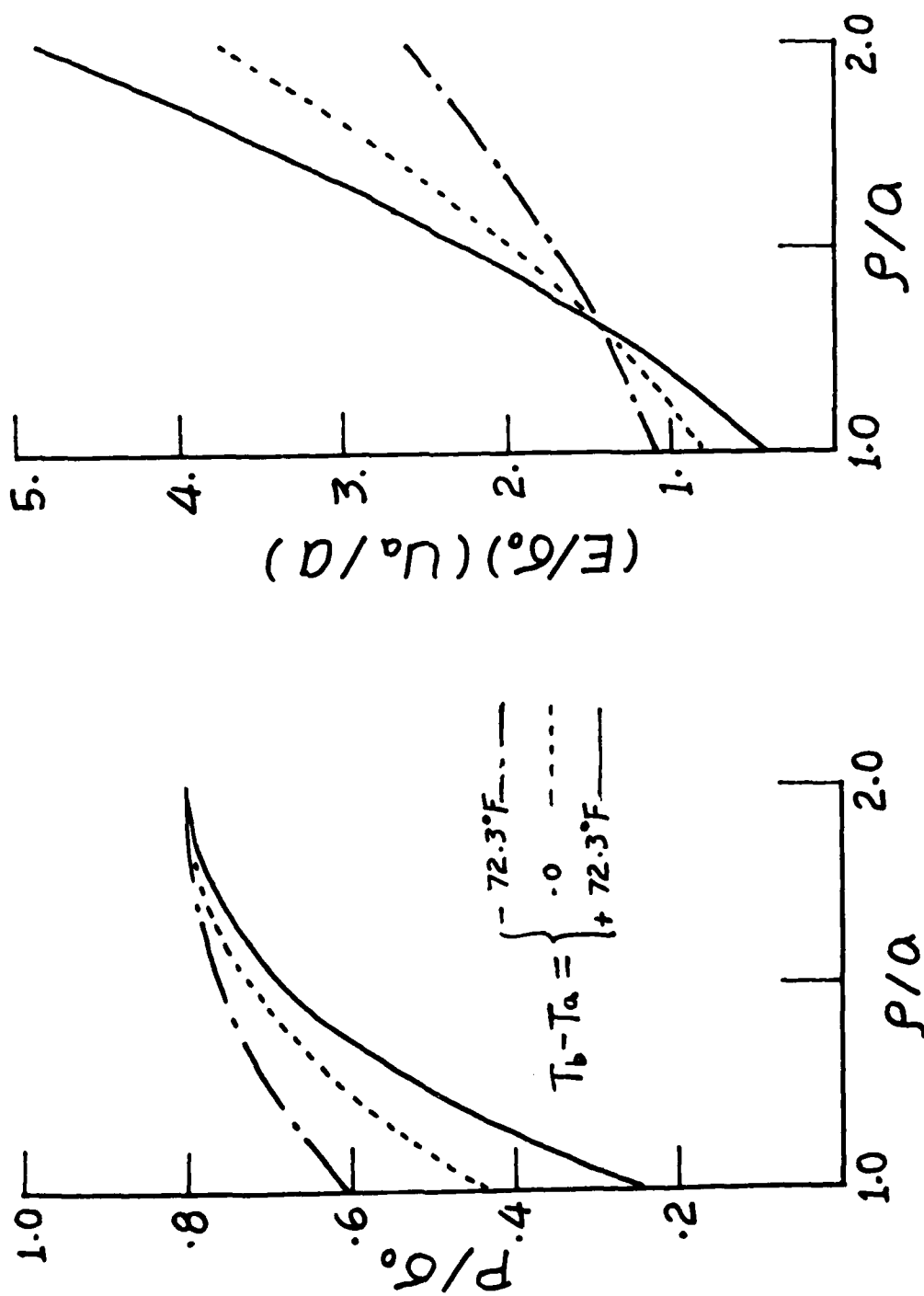


Figure 1. Internal pressure and displacement as functions of elastic-plastic interface in a closed-end tube. (a) Internal pressure; (b) Internal displacement.



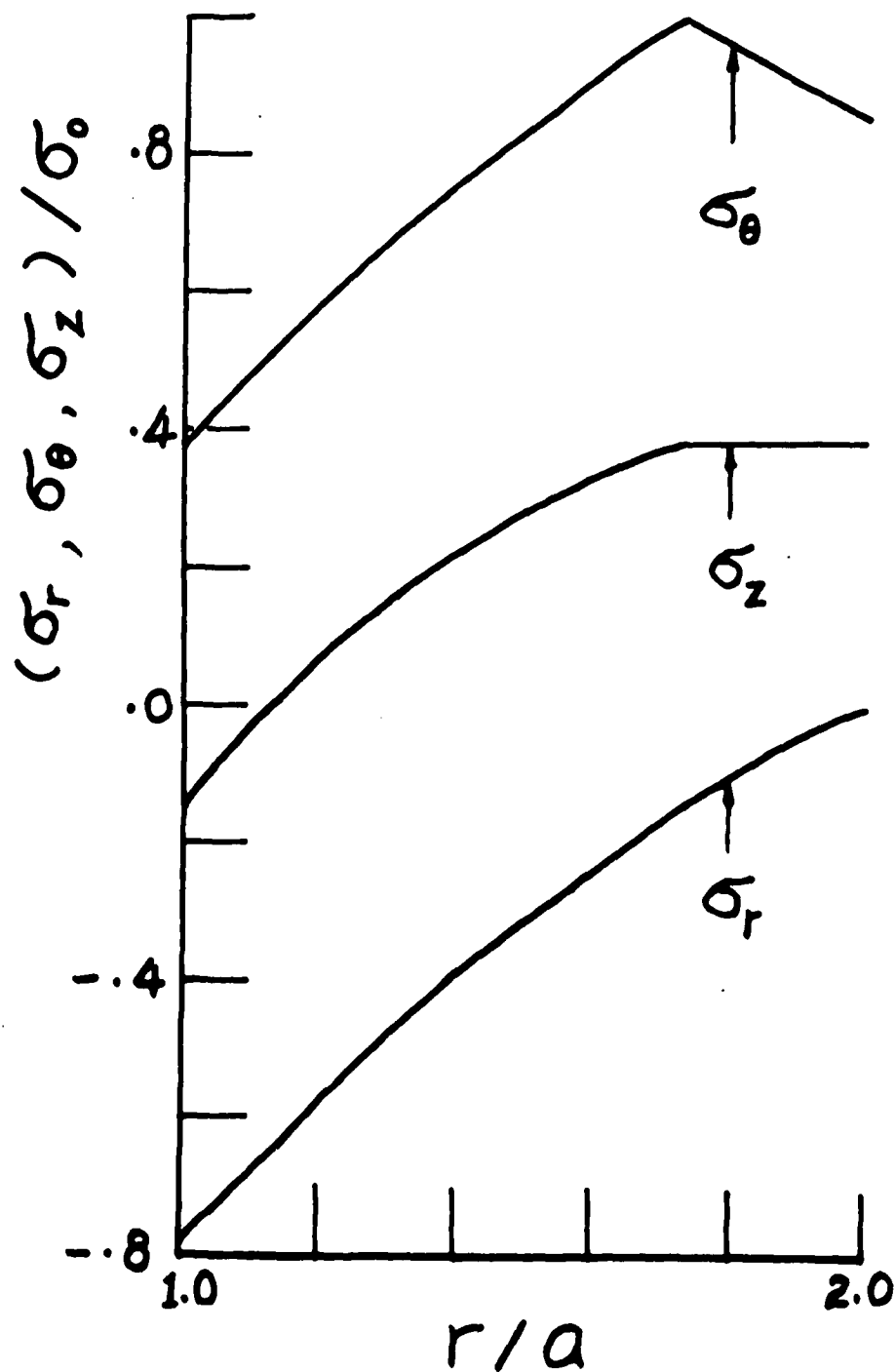


Figure 2. Stresses in a closed-end tube subjected to internal pressure ( $p/a = 1.73$ ;  $\theta = 0$ ).

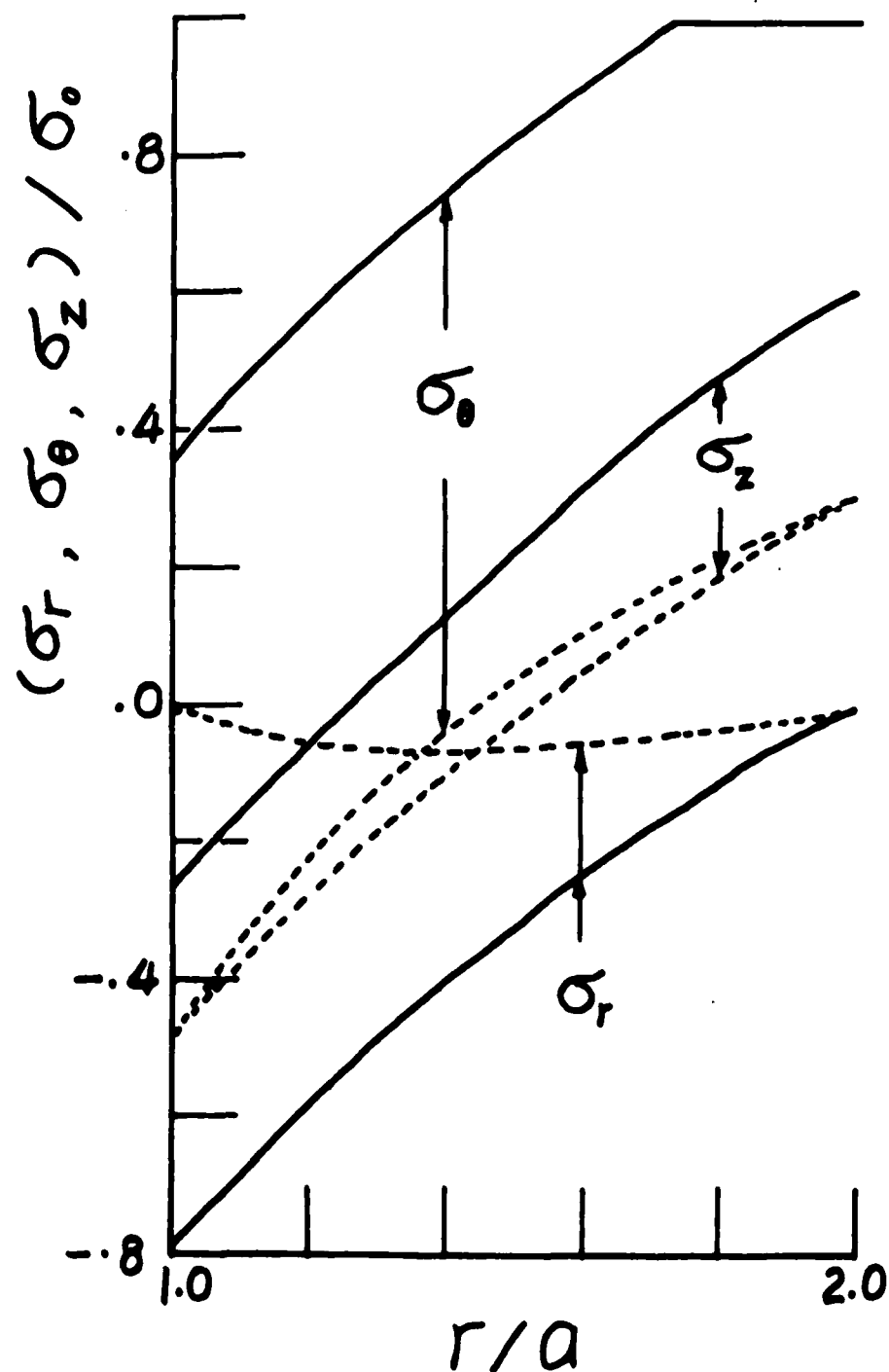


Figure 3. Stresses in a closed-end tube subjected to internal pressure and temperature gradient ( $p/a = 1.73$ ;  $\theta = -1/2$ ).

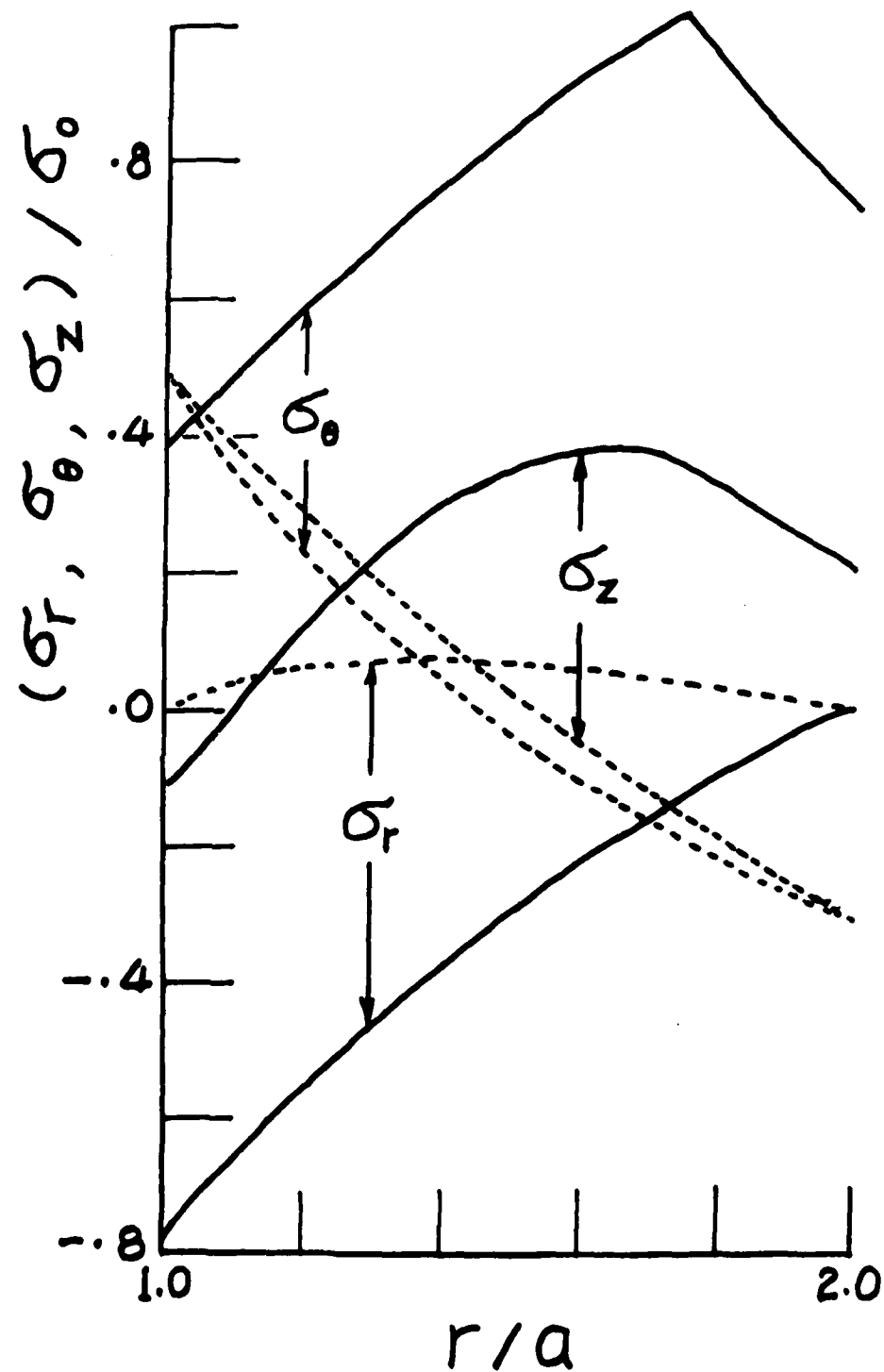


Figure 4. Stresses in a closed-end tube subjected to internal pressure and temperature gradient ( $\rho/a = 1.73$ ;  $\theta = 1/2$ ).

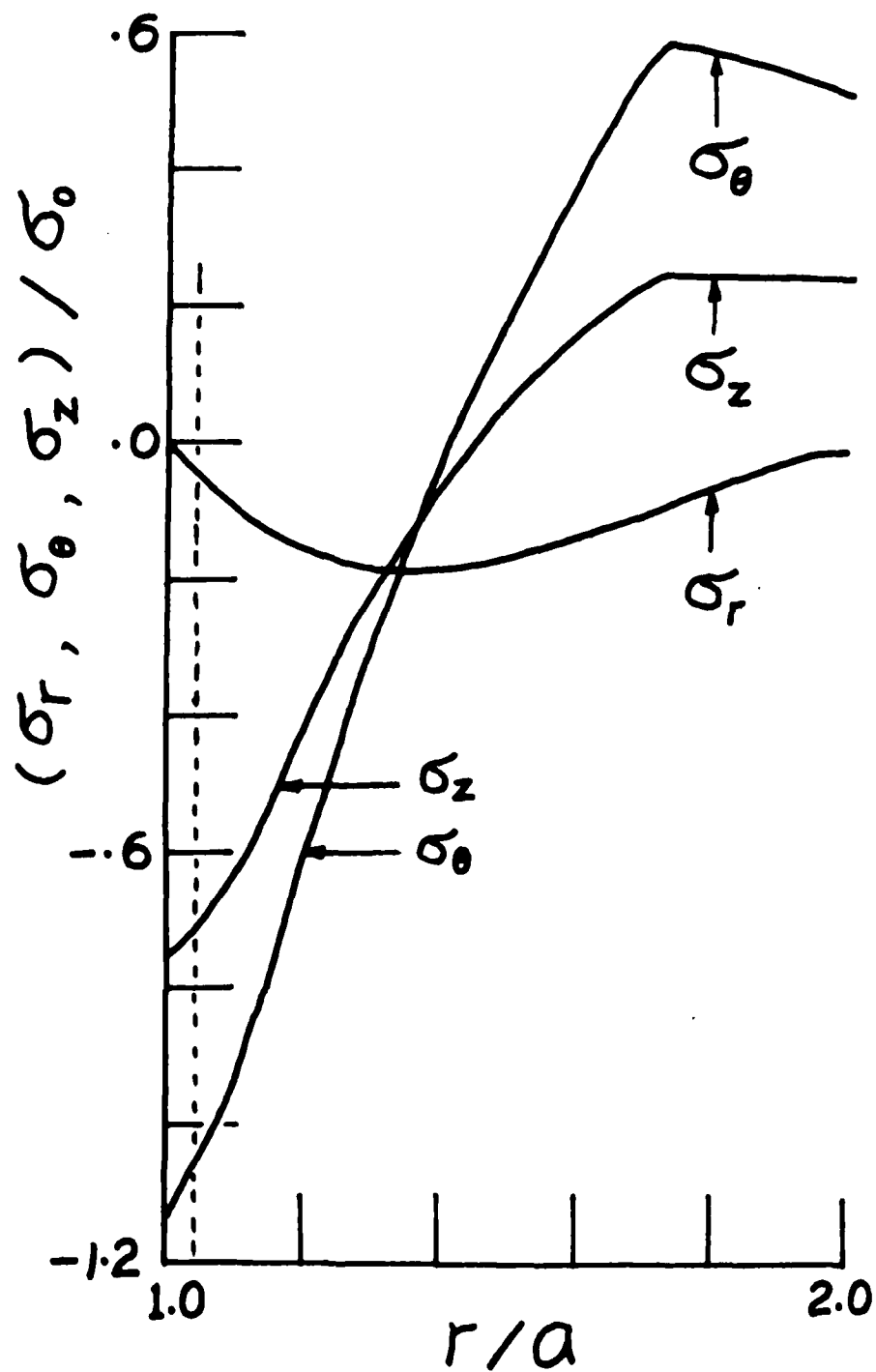


Figure 5. Residual stresses in a closed-end tube, unloading from Figure 4,  $1.04 < \rho/a < 1.05$ .

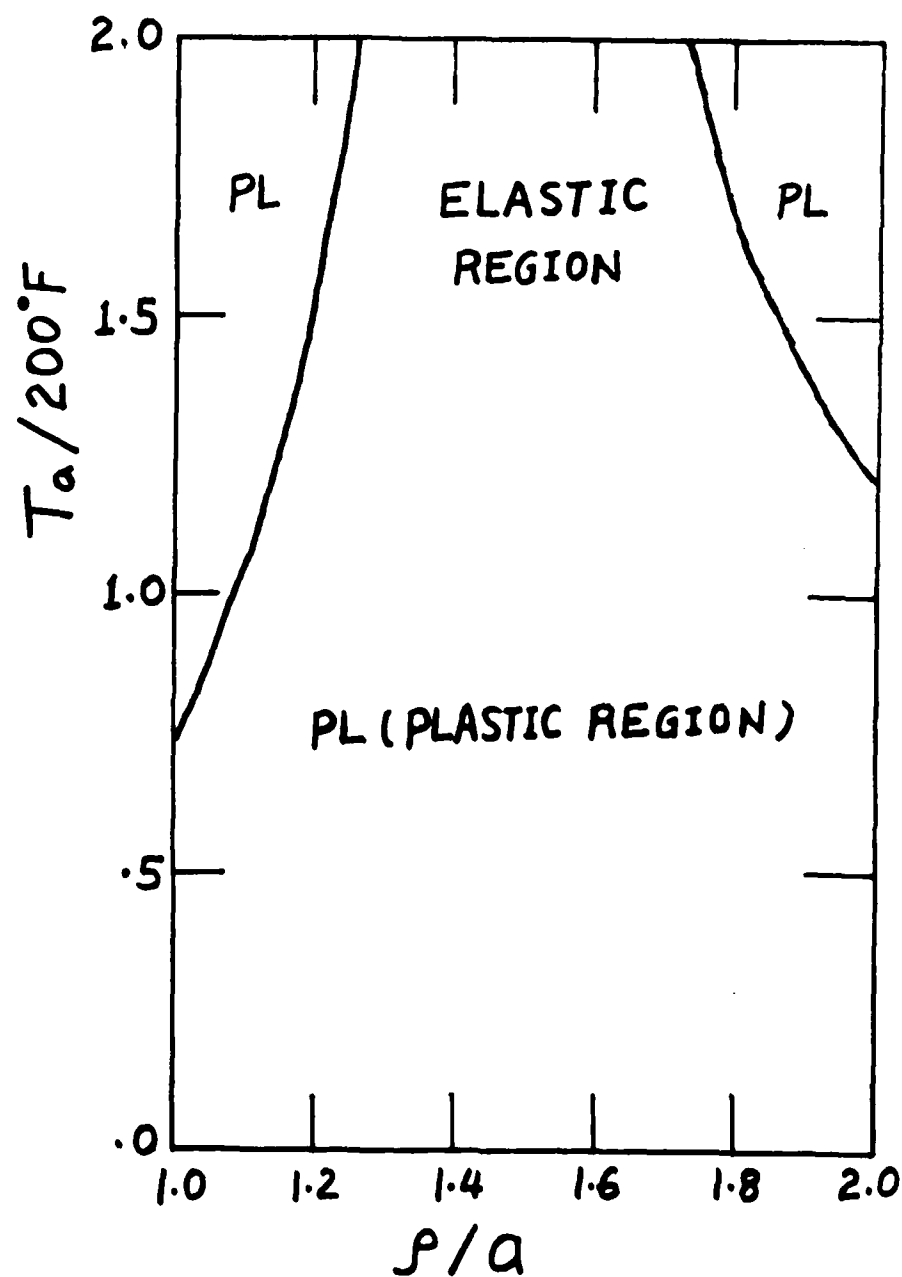


Figure 6. The relation between internal temperature and elastic-plastic interface,  $T_b = 0$ .

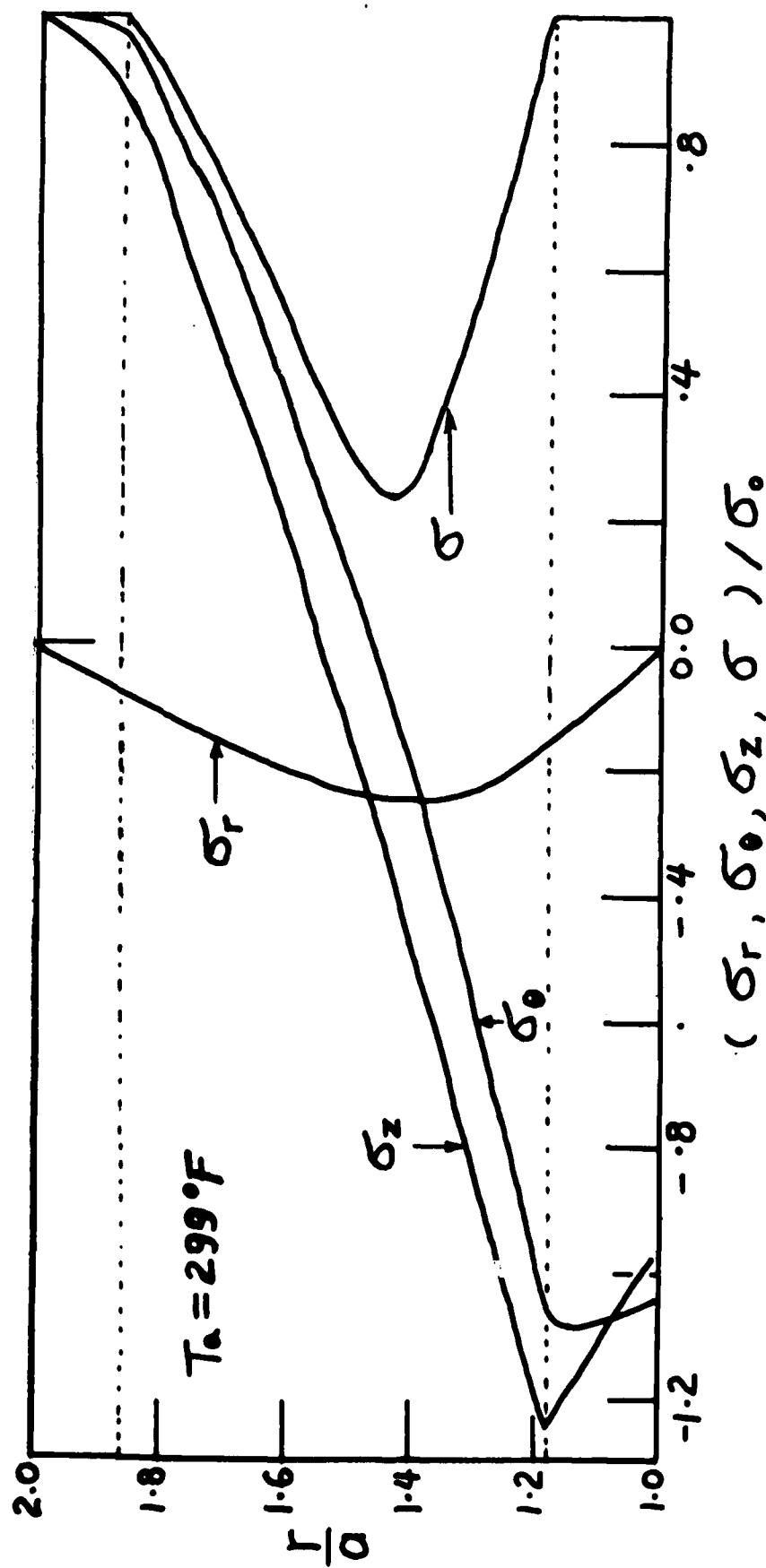


Figure 7. Stresses in a closed-end tube subjected to temperature gradient,  $T_b = 0$ .

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